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MECHANICS OF ELASTIC PERFORMANCE OF TEXTILE MATERIALS

GRAPHICAL ANALYSIS OF FABRIC GEOMETRY

by
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DEPARTMENT OF THE ARMY

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FOREWORD

This report is an outgrowth of the research program sponsored by the QMC on fiber properties and on the translation of these properties into yarn and fabric structures. The graphical technique, which was developed during the course of this work, provides a relatively simple means of applying the geometrical analyses of fabric structure developed by Dr. F. T. Peirce over fifteen years ago. The graphs may be utilized to supplement and extend the work on the interrelationship of fiber and yarn properties which has been published previously in Textile Series Reports No. 60 and No. 62 and to integrate this information as a basis for predicting the mechanical performance of woven fabrics.

It is gratifying that increasing use is being made of the work of Peirce by textile technologists to facilitate the design and to understand the performance of textile fabrics. Even greater use of his work would result by the elimination of the tedious work involved in the solution of his equations and the computations that are necessary to derive his basic geometrical parameters.

Through the use of the graphs which are described in this report much of the computational work is eliminated. With basic knowledge of the texture, yarn counts and crimp, the complete geometry of the fabric can be obtained and the changes in the geometry due to shrinkage, swelling, flattening, and crimp interchange can be predicted.

It is hoped that the publication of this report provides additional stimulation to the textile technologist to apply the findings of Peirce to the improvement of the design and functional performance of textile fabrics.

S. J. KENNEDY
Research Director
for
Textiles, Clothing and Footwear

June 1952

ABSTRACT

Graphs have been developed to simplify the study of fabric geometry. A simultaneous plot of the mathematical relations developed by Peirce aids in visualizing dimensional changes in fabrics and gives accurate values of the various parameters without lengthy calculations and interpolations from tables.

The use of the graphs is discussed as they apply to various types of weaves, fibers, and their dimensional changes.

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INTRODUCTION

The modern developments in new fabric treatments and weaving of new fibers have not been aided as much as they should have been by a well-developed theory of the form and changes therein of fibrous structures. The cross-sectional swelling of a rayon fiber can be determined accurately in the laboratory, but by the time this fiber is twisted into a yarn, which is then woven into a complex geometric form, it is practically impossible to follow rigorously the chain of events which relate this simple change in the fiber to the dimensional changes observed in the fabric. The endless variety of fibers, twists, and weaves makes the job almost impossible of solution by empirical methods. The only alternative is the development of a theory of ideal behavior, so that each specific case may be reduced to a simple study of deviations from the ideal.

The brilliant mathematical analyses of Peirce[1,2] have laid the groundwork for this theory of the ideal. However, considerable calculation and lengthy interpolations from tables are required to make use of his work. Peirce recognized this difficulty and suggested the use of graphs for particular relations. This idea has been carried a little further in the present study; one graph is presented herein which depicts all of the relations of the ideal plain-weave geometry, much in the manner of a psychrometric chart or Mollier diagram. The use of the graphs is described in the following section, while the details of derivation and method of construction are given in the Appendix. Peirce's nomenclature is used throughout.

USE OF THE GRAPHS

The main graph is shown in Figure 1, and the auxiliary graph in Figure 2. The main graph depicts the geometrical relations of the plain weave, while the auxiliary graph gives related information in terms of thread count and yarn number.

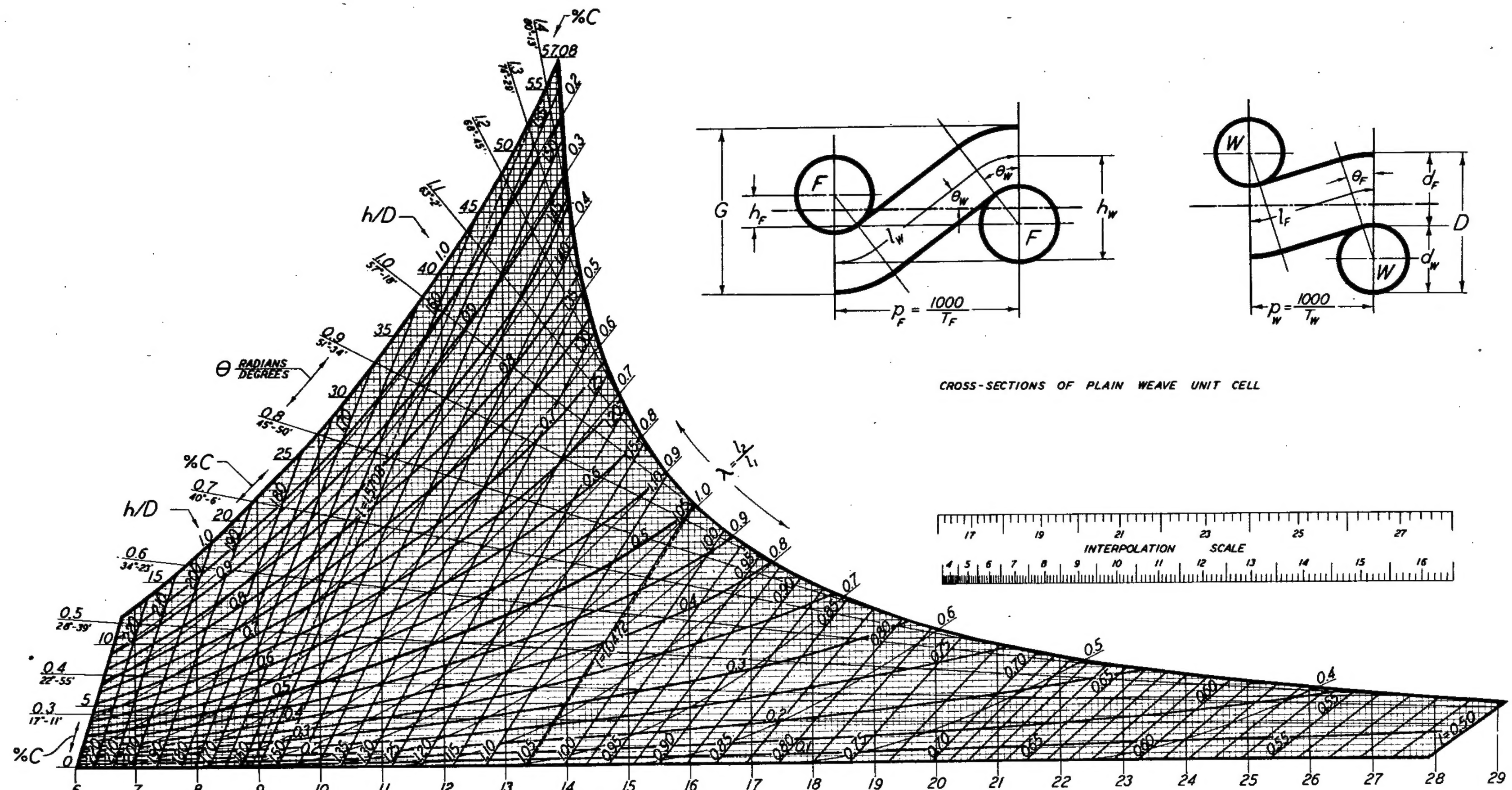
Description of the Main Graph

In Figure 1 any plain-weave fabric is fully defined by a pair of two related points which can be located on the graph to give numerical values for ten of the eleven variables (D is the eleventh) which define a fabric structure. This is made possible

FIGURE I

GEOMETRICAL RELATIONS OF THE PLAIN WEAVE

BASED ON EQUATIONS GIVEN BY F. T. PEIRCE



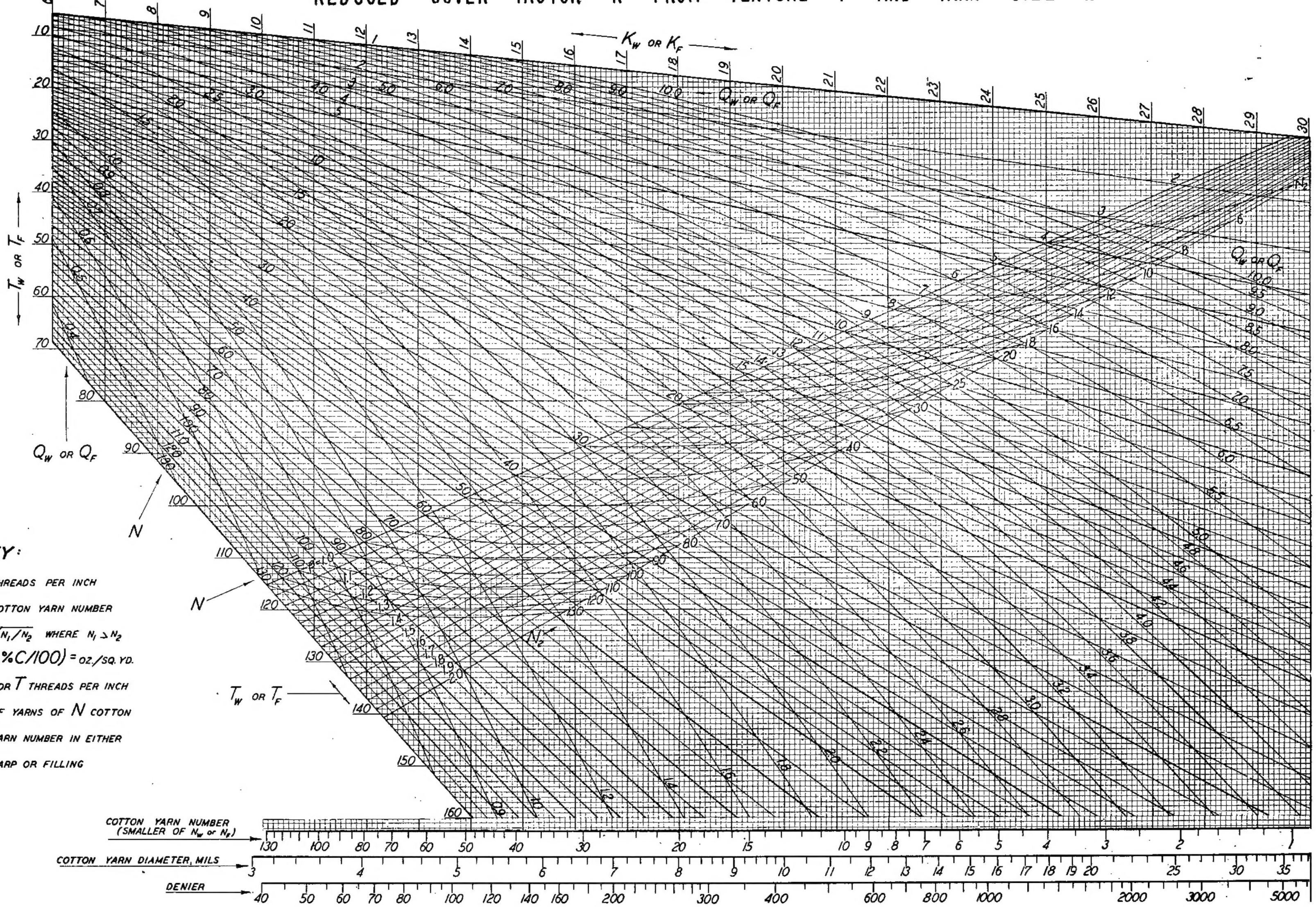
CROSS-SECTIONS OF PLAIN WEAVE UNIT CELL

$K_W = T_W/\sqrt{N} = 0.01395 D \times T_W$

$K_F = T_F/\sqrt{N} = 0.01395 D \times T_F$

FIG 2

REDUCED COVER FACTOR K FROM TEXTURE T AND YARN SIZE N



KEY:

T = THREADS PER INCH

N = COTTON YARN NUMBER

$\beta = \sqrt{N_1/N_2}$ WHERE $N_1 \geq N_2$

$Q(1+\%C/100)$ = OZ./SQ. YD. FOR T THREADS PER INCH OF YARNS OF N COTTON YARN NUMBER IN EITHER WARP OR FILLING

by the superposition of several sets of curves and lines on a background set of rectangular coordinates relating % crimp to reduced cover factor. From the location of one of the points on the graph, it is possible to read off directly the reduced cover factor for the filling yarns, K_f, and the following values for the warp yarns: the % crimp, $\%C_w$; the fractional length of yarn between two filling yarns, l_w/D; the fractional maximum amplitude, h_w/D; and the maximum angle of inclination, θ_w. The other point of the pair locates the value of the reduced cover factor for the warp yarns, K_w, and the following values for the filling yarns: the % crimp, $\%C_f$; the fractional length of yarn between two warp yarns, l_f/D; the fractional maximum amplitude, h_f/D; and the maximum angle of inclination, θ_f. Thus, it might be said that one point represents the geometry of the warp yarns versus the spacing of the filling yarns, K_f, while the other point represents the geometry of the filling yarns versus the spacing of the warp yarns, K_w. The two points are located on the graph in relation to one another by the simple fact that

$$h_w/D + h_f/D = 1.0. \qquad (1)$$

The set of curves on the graph (Figure 1) giving values of h/D are those curving upward to the right. The heavy line for h/D = 0.5 separates the upper part of the graph from the lower; one of the two points defining a fabric must lie above this line, and the other below it, except for the special case when both points are directly on it. The upper left boundary of the graph is the line h/D = 1.0; when one of the points is on this line, the other must be on the base line at h/D = 0.

The straight lines inclined to the right in Figure 1 give values for l/D. In depicting the interchange of crimp between warp and filling yarns of a fabric, the two points defining the fabric must move along these straight lines, one point up and the other down (keeping $h_w/D + h_f/D = 1.0$). One of the points must lie on or to the left of the line marked l = 1.5708, in order to allow the other set of yarns to straighten out completely, a complete straightening represented by the point being located on the base line. However, if both points lie to the right of the line l = 1.5708, then neither set of yarns can ever assume a straight-line configuration, and the minimum crimp possible in either set of yarns is governed by the condition in which they are jammed against the other set of yarns. This condition occurs where the top of the l/D line terminates at the "jam line," which is the upper right-hand boundary of the graph along which various values of λ have been laid out.

$$\lambda = l_2/l_1 \quad (\text{where } l_2 < l_1) \qquad (2)$$

The ratio is always taken so that $\lambda \leq 1.0$, regardless of whether the length of yarn between crossovers, l, for warp or for filling must be placed in the numerator. The value of λ expresses the

"squareness" of a fabric—i.e., values of λ near 1.0 indicate square interstices, and the interstices become more and more oblong as λ decreases. The λ lines are laid on the graph in pairs, one line referring to the point for the warp yarns and the other line referring to the point for the filling yarns. Thus, if the point for the filling yarns lies at maximum crimp on the jam line (e.g., point 8, Figure 3), then the point for the warp yarns must lie at minimum crimp at the intersection of the line and the l_W/D line (e.g., point 12, Figure 3). The λ lines are especially helpful in the design of fabrics, since yarns do not slip along one another at the crossovers, and, hence, the setup in the fabric at the loom remains the same for the life of the fabric.

One more set of lines on the main graph remains to be discussed. The straight lines inclining upward to the left give values of θ, the angle of maximum inclination of a yarn to the plane of the fabric. The value of this angle is of little use in most fabric-geometry problems, except where it might be used in the calculation of stress translations in tensile-strength computations. The angle θ is important in the basic theory of fabric geometry since it occurs in four of the seven simultaneous equations which link the eleven variables defining the plain weave.

Examples of Use of the Main Graph

There are three major changes in shape of the unit cell outlined by Peirce which cause concomitant changes in fabric dimensions: (1) crimp interchange, (2) yarn swelling, and (3) yarn flattening. The geometry of these various shapes is shown in Figure 3 in relation to the points on the main graph which depict them (points 8-15). In addition, Figure 3 shows the shapes of the unit cell at various other "limiting" points on the graph (points 1-7).

The assumptions and calculations necessary to construct the examples are given in Table I. A tight-weave fabric was chosen in order to demonstrate the use of the jam line in defining limiting values. For a fabric with neither yarn jammed, it is necessary to fix four of the eleven variables which define any plain-weave fabric; if one yarn is jammed, only three of the values are needed for definition of the fabric structure; if both yarns are jammed, only two values need be known. Actually, in the last case, the geometry of the structure can be defined by the ratio $\lambda = l_2/l_1$, but the scale will not be fixed unless one other value is fixed, such as one of the l's. The variable D can be regarded as the scale factor which fixes linear fabric dimensions such as threads per inch, thickness, yarn diameter, etc. Finding the finite value of D and these attendant fabric dimensions is the function of the

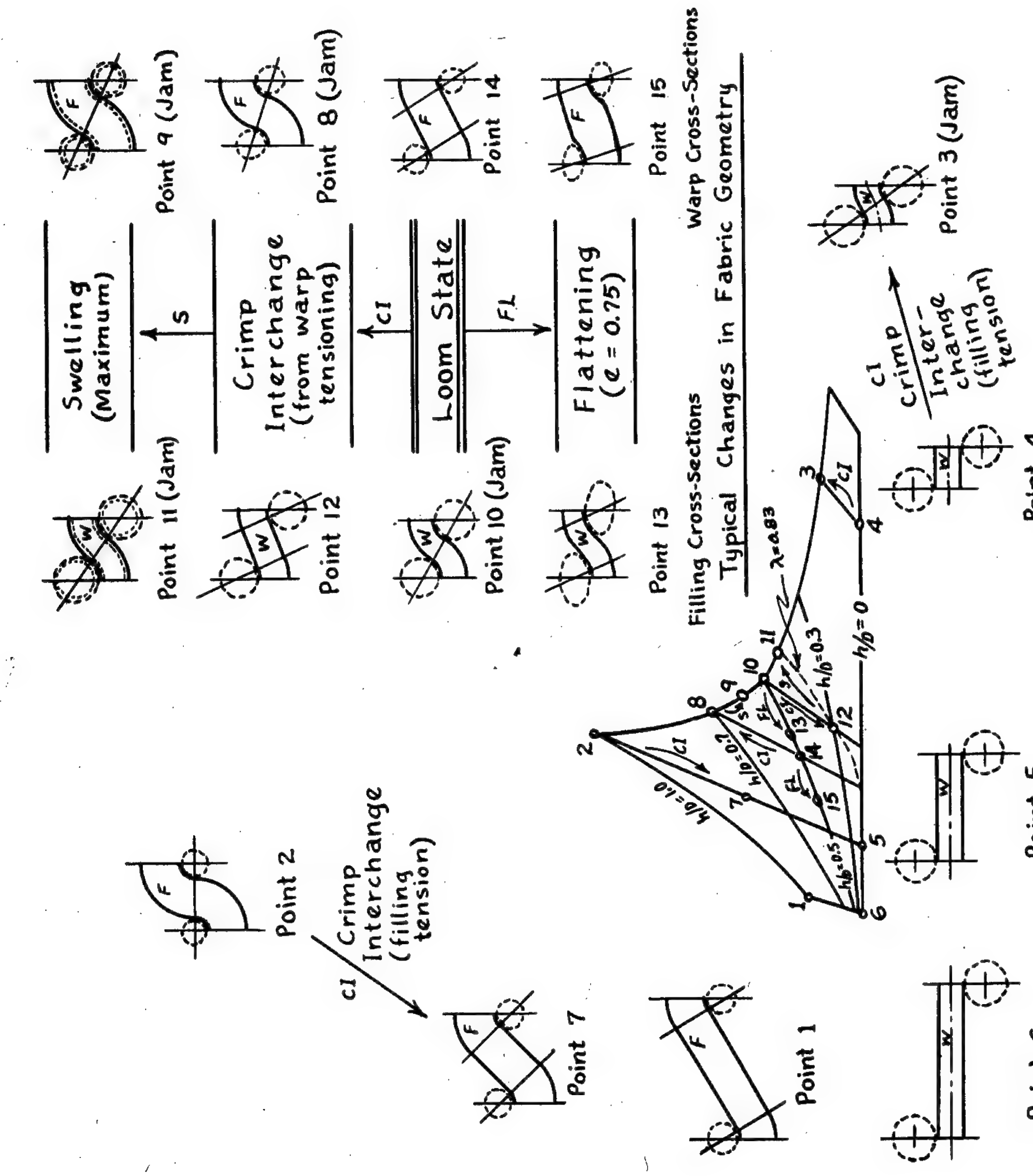


Fig. 3. Geometry of Various Points on Main Graph.

auxiliary graph, and the examples shown in Table I in terms of D are carried over into Table V to demonstrate the use of the auxiliary graph in defining the physical dimensions of a fabric.

The two l/D values chosen for the loom-state example in Table I were selected rather arbitrarily as follows: Since the loom-state fabric has minimum filling crimp, the warp point must be on the jam line for a tight-woven fabric, and the point $l_w/D = 1.047$ was selected since $h_w/D = 0.500$ at this point. This is a central

point of the graph and it is useful to show a drawing of the configuration at this point. The $l_f/D = 1.265$ was selected since it hits the jam line at $h_f/D = 0.700$, which is a convenient intermediate point to show in the drawing. After the assumption of these initial values, the changes in geometry which follow can be calculated as shown in Table I without further assumptions except as to whether the yarns remain round or are permitted to flatten.

The diameters of yarns shown in the examples in Figure 3 were chosen so that $\beta = 1.5$—that is, $d_f = 1.5\ d_w$, or $\sqrt{N_w/N_f} = 1.5$. However, it should be pointed out that the fabric geometry and changes therein would be the same whatever β was chosen. As long as the sum $d_1 + d_2 = D$ remains constant, the geometry is unchanged.

In the examples of crimp interchange, swelling, and flattening in Table I, it can be seen that the basic assumption is that yarns do not slip at crossovers—that is, the values of l_w and l_f remain constant throughout all changes of geometry. This is one of the basic concepts of fabric dynamics.

In the example of flattening given in Table I, the assumption was made that the final state of crimp was determined by the h/D's remaining constant. As Peirce pointed out, it is always difficult to predict the state of crimp balance remaining in a fabric after a change in D (either flattening of yarns or partial, but not maximum, swelling; crimp balance is defined by λ for maximum swelling). Of course, reasonable assumptions as to state of crimp balance can be made once the cause of the change in D is known. For instance, for partial swelling, the crimp balance tends towards the so-called "normal crimps," as defined by the h/D's which intersect the jam line at the λ value of the fabric. Thus, if the sample fabric were dried back to the original loom-state yarn diameters after swelling, the normal crimps would be given as follows: normal $\%C_w = 10.7$ at intersection of $h_w/D = 0.415$ and $l_w/D = 1.047$ ($K_f = 14.8$); and normal $\%C_f = 16.8$ at intersection of $h_f/D = 0.585$ and $l_f/D = 1.265$ ($K_w = 12.9$). The crimp balance to be expected in the case of flattening by calendering would be defined, as Peirce pointed out, by equating the sum of warp yarn diameter plus its amplitude, h_w, to the sum of filling yarn diameter plus its amplitude, h_f: thus, $d_w + h_w = d_f + h_f$. If one set of yarns is sized or is held under tension, that set will probably remain round, and all of the flattening will occur in the opposite set of yarns.

The shrinking or stretching of a fabric is governed by three effects: (1) crimp interchange, (2) change in yarn diameter, and (3) change in yarn length, l. The last factor can usually be ignored except in the case of very extensible yarns. Both (1) and (2) often occur simultaneously, which complicates an analysis

Table I

Calculations For Typical Changes In Fabric Geometry Shown In Figure 3

Configuration	Points for warp yarn geometry	Points for filling yarn geometry
Loom state*	Point 10 (Jam)	Point 14
Assume:	$l_w/D = 1.047$	$l_f/D = 1.265$
Calculate:		$h_f/D = 1 - h_w/D = 0.500$
Read from graph:	$h_w/D = 0.500$ $\%C_w = 20.9$ $K_f = 16.07$	$\%C_f = 10.6$ $K_w = 12.20$
Calculate:	$\lambda = 1.047/1.265 = 0.83$	
Crimp interchange**	Point 12	Point 8 (Jam)
Calculate:	$h_w/D = 1 - h_f/D = 0.300$	
Read from graph:	$\%C_w = 5.0$ $K_f = 14.00$	$h_f/D = 0.700$ $\%C_f = 32.8$ $K_w = 14.65$
Calculate:	$S_w = 1 - \frac{120.9}{105.0}$ $= -0.151$ shrinkage $= 0.151$ stretch in length	$S_f = 1 - \frac{110.6}{132.8}$ $= 0.167$ shrinkage in width
Swelling (maximum)***	Point 11 (Jam)	Point 9 (Jam)
Read from graph:	$l_w/D = 0.95$ $\%C_w = 16.80$ $h_w/D = 0.415$ $K_f = 17.17$	$l_f/D = 1.14$ $\%C_f = 25.8$ $h_f/D = 0.585$ $K_w = 15.37$
Calculate:	$S_w = 1 - \frac{105.0}{116.8}$ $= 0.105$ shrinkage in length	$S_f = 1 - \frac{132.8}{125.8}$ $= -0.056$ shrinkage $= 0.056$ stretch in width
	$\frac{(\text{new } D)}{(\text{old } D)} = \frac{(\text{old } l/D)}{(\text{new } l/D)}$; since l is constant. $\therefore \frac{(\text{new } D)}{(\text{old } D)} = \frac{1.047}{0.95} = \frac{1.265}{1.14} = 1.105,$ or, each yarn swelled 0.105 in diameter.	
Flattening****	Point 13	Point 15
Calculate:	$(\text{new } l_w/D) = \frac{1.047}{0.75} = 1.396$	$(\text{new } l_f/D) = \frac{1.265}{0.75} = 1.687$
Read from graph:	$\%C_w = 8.0\%$ $K_f = 10.75$	$\%C_f = 5.0$ $K_w = 8.66$
Calculate:	$S_w = 1 - \frac{120.9}{108.0}$ $= -0.119$ shrinkage $= 0.119$ stretch in length	$S_f = 1 - \frac{110.6}{105.0}$ $= -0.056$ shrinkage $= 0.056$ stretch in width

* This configuration illustrates the loom state because crimp is minimum in filling. Assume round yarns.

** Assume warp tensioning to produce minimum crimp in warp, keeping yarns round. Solution: Keep l/D's constant; points move along l/D lines until filling hits jam line.

*** Assume yarns remain round, swell equally to minimum density, or maximum D, at which both yarns jam against each other. Solution: Keep l's constant, increase D to give minimum possible l/D's as read from jam line at $\lambda = 0.83$.

**** Assume yarn diameter which is vertical to cloth plane is reduced 25%—that is, flattening coefficient $e = 0.75$. Solution: Keep l's constant, decrease D so that new $D = 0.75$ (old D). Assume h/D's constant to determine new state of crimp balance.

or prediction of a fabric change. The two effects can be studied separately by use of the graph: the changes in fabric length and width for each type of structure change are shown in Table I. The changes in length or width can be calculated from either the change in crimp or the change in threads per inch.

$$S_w = 1 - \frac{100 + (\%C_w)o;}{100 + (\%C_w)_s} \qquad S_f = 1 - \frac{100 + (\%C_f)o;}{100 + (\%C_f)_s} \tag{3}$$

$$S_w = 1 - (T_f)_o/(T_f)_s \qquad S_f = 1 - (T_w)_o/(T_w)_s; \tag{4}$$

where S_w = warpwise, or length, shrinkage; S_f = fillingwise, or width, shrinkage; $\%C_w$ = % crimp in warp yarns; $\%C_f$ = crimp in filling yarns; T_w = warp threads per inch; T_f = filling threads per inch; o = original fabric structure; s = shrunk fabric structure.

If the calculated value of S_w is negative, then the fabric has stretched instead of shrunk. Calculations of shrinkages from crimp changes are given in Table I. Calculations using values of T in equation (4) give the same values for shrinkage as are obtained by using values of crimp; this method is useful in checking the accuracy with which %C's and K's are read from the graph. It should be pointed out that the change in K upon shrinking reflects a change in both T and D, and when T's for the shrunk cloth are calculated from K's for the shrunk cloth, the change in D must be taken into account by using the value of D for the shrunk cloth.

Locating D on the Main Graph

The geometrical analysis of any particular fabric consists merely of locating the two points representing that fabric on the main graph. In order to do this, it is necessary to obtain the %C's and K's for each set of yarns. The value of K is obtained from T (threads per inch) and the proper value of D (the sum of diameters, or amplitudes, in mils) by the expression $K_w = 0.01395\ DT_w$. However, flattening of yarns is usually present to an unknown degree, and the value of D cannot be calculated from the sum of the diameters of the individual yarns because these are obtained on the assumption that the yarns are round. The proper value of D_f for flattened yarns can be obtained from the main graph by the simple procedure shown in Table II and illustrated in Figure 4. The procedure is based on the use of Peirce's approximate equation for amplitude:

$$h_1 = \frac{136\sqrt{\%C_1}}{T_2} \tag{5}$$

The sum

$$h_1 + h_2 = D_f \tag{6}$$

Table II

Example of Locating Precise D_f

	W	X	F	
Threads/in., T	72	X	53	Given
Crimp, %C	10.8		16.6	
$136\sqrt{\%C_1}/T_2 = h_1$	8.4			Approximate
$136\sqrt{\%C_2}/T_1 = h_2$			7.7	
$h_1 + h_2 = D_f$		16.1		
$.01395(T_1)16.1 = K_1$	16.2			
$.01395(T_2)16.1 = K_2$			11.9	
Read h/D's from graph:	0.520		0.450	
Adjust sum h/D's = 1.0, or (h/D)/0.970 gives:	0.536		0.464	
Using adjusted h/D's read K's from graph as in Figure 4:	15.7		11.6	Precise
$K/.01395T = D'_f$		15.7		
$(0.536)(15.7) = h'_1$	8.4			
$(0.464)(15.7) = h'_2$			7.3	

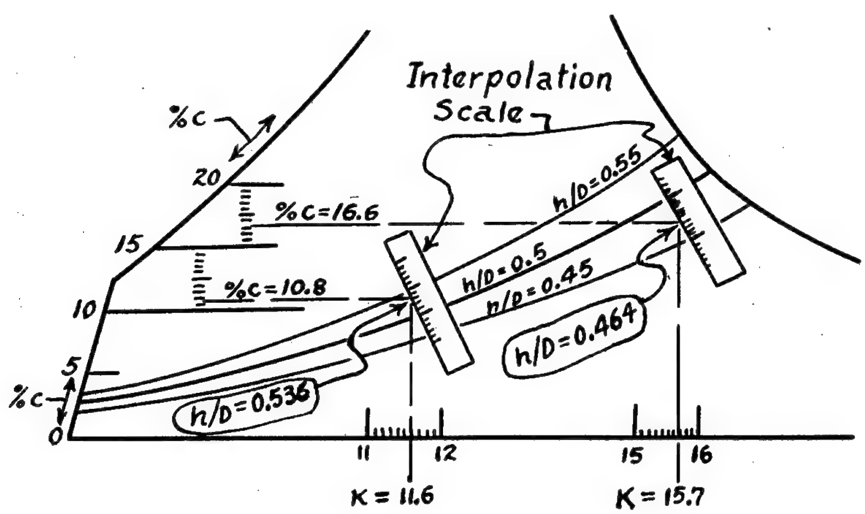


Fig. 4. Location of D_f

then gives an approximate value of D_f, allowing calculation of K's. . These values of K can, however, be considered as precise values even though calculated from approximate h's if one assumes that h/D's are independent of D--that is, the error is distributed proportionately between the two values of h. Of course, if it is known that all of the flattening occurs in one set of yarns only, such as would occur with hard sized-warp yarns, all of the error can be proportioned to the soft yarns, thus changing the h/D values. In making such assumptions, one must use great care in determining proper values for % crimp, since extensible yarns, or stiff yarns with crimp set into them, will give slightly erroneous experimental results, and small variations in crimp have a rather large influence on the results. It will nearly always be possible to determine D_f in this manner, with a precision of about 1% or better, since K can easily be read to the nearest 0.1.

This procedure for obtaining the precise value of D_f is offered as something of an improvement over the method given by Peirce, using the following equation:

$$D_f = \frac{p_2\sqrt{2C_1} + p_1\sqrt{2C_2}}{1 + 0.20\,(C_1 + C_2) - (S_1 + S_2)}\,, \qquad (7)$$

where $p = 1000/T$ = thread spacing, and S = correction factor to be read from a graph of $\sqrt{\%C}$ vs. D/p. It is necessary to go through the above equation once without using the S's in order to get an approximate D with which to calculate one scale for the graph. Thus, a trial-and-error method is necessary. It is not simple in the present procedure to apportion all of the error to one set of yarns.

A further example of locating a fabric on the main graph is illustrated in Figure 5, which depicts the various fabric structure changes in an example given by Peirce. Various weights were hung on wet strips of fabric and Peirce then analyzed the structures after drying them under tension. The calculations for location of the loom-state sample are shown in Table III, and the values of K and %C for the various stretched samples are shown in Table IV. The lines for the average 1/D values computed by Peirce are shown in Figure 5. It is interesting to note that the initial warp tension produced almost pure crimp interchange, as illustrated by movement of the points parallel to the constant 1/D lines, while the initial filling tension produced almost pure flattening, as illustrated by movement of the points parallel to the constant h/D lines.

Description of the Auxiliary Graph

The terms usually used to describe a fabric are texture, T (threads per inch), yarn number, N (or cotton count or denier),

FIG. 5

DIMENSIONAL CHANGES IN A CANVAS

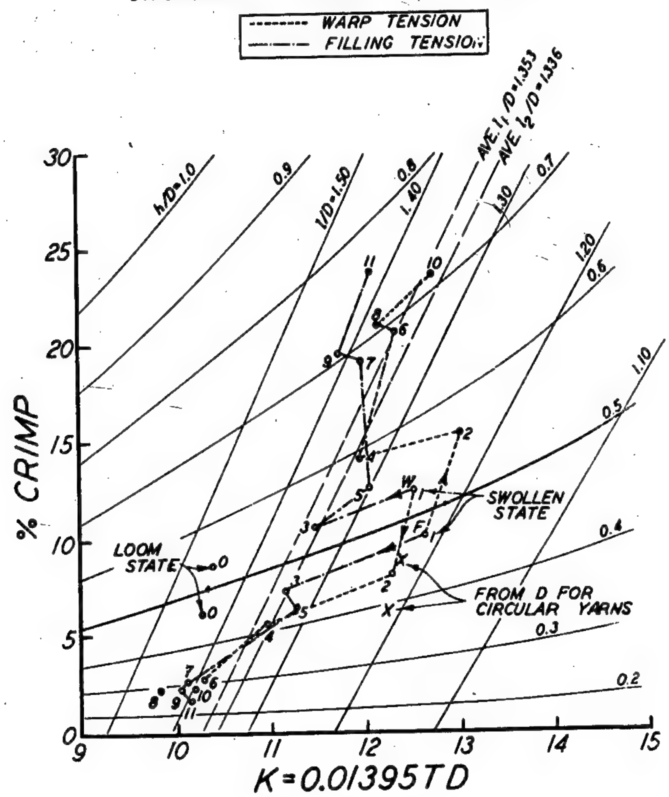


Table III

Locating Precise D_f for Loom-State Sample

	W	X	F	
Threads/in., T	32.4	X	32.8	Given
Crimp, %C	8.7		6.2	
Yarn Nos., N_1, N_2	6.7		6.3	
$36/N_1 = d_1$	13.8mils		14.4 mils	
Rouhd-yarn sum, D		28.2 mils		
Reduced yarn No., N		6.5		
$136\sqrt{\%C_1}/T_2 = h_1$	12.2 mils		10.4 mils	Approximate
$h_1 + h_2 = D_f$		22.6 mils		
h/D_f	0.540		0.460	
Read from graph, K	10.3		10.4	
$K/0.01395T = D'_f$		22.8 mils		Precise
$(22.8/22.6)h = h'$	12.3 mils		10.5 mils	

Flattening coeff., e = 22.8/28.2 = 0.81

Thickness, G = 13.8 + 12.3 = 26.1 mils

Table IV

Points for Graph from Peirce's Data for "Dimensional Changes in a Canvas"

Cloth	Load* W	Load* F	D/p_1	D/p_2	K_1**	K_2**	%C_1	%C_2
0	Loom		.737	.747	10.29	10.41	8.7	6.2
1	Shrunk		.908	.898	12.65	12.51	12.5	10.3
2	.5		.934	.882	13.01	12.30	8.3	15.5
3		.5	.801	.824	11.17	11.48	10.8	7.4
4	1		.860	.787	11.98	10.97	5.6	14.1
5		1	.811	.865	11.30	12.06	13.6	6.5
6	2.5		.885	.739	12.33	10.30	2.8	20.7
7		2.5	.727	.860	10.13	11.99	19.3	2.7
8	5		.870	.705	12.12	9.84	2.3	21.0
9		5	.722	.840	10.08	11.71	19.6	2.3
10	10		.913	.734	12.72	10.22	2.4	23.6
11		10	.730	.866	10.18	12.08	23.8	1.7

* Load (lbs.) on 2 in. x 18 in. wet strip of fabric.
** K = 13.95 D/p.

and weight, W (ounces per square yard). These terms are not available directly from the main graph, but they may be calculated by means of a few simple relations. These relations have been reduced to graphical form in Figure 2, the auxiliary graph. This has been done despite the fact that the equations involved are quite simple; the graphical approach has the advantage over mere calculation of particular values in that limits are always clearly depicted.

It is apparent from the equations shown along the base of the main graph (Figure 1) that T is easily found from K if D is known, since

$$K_w \doteq 0.01395\, DT_w. \tag{8}$$

It is developed in more detail in the following section that D can be stated in terms of N, the reduced yarn number, as follows:

$$D = 1/.01395\sqrt{N} = (2)\ (35.85)/\sqrt{N}. \tag{9}$$

Hence:

$$K_w = T_w/\sqrt{N} \tag{10}$$

In Figure 2 various sets of lines have been superposed on a background set of rectangular coordinates relating T, threads per inch (along the left side), to K, reduced cover factor

(along the top). Various values of N, reduced yarn number, are given on the set of straight lines sloping downward to the right. Any point along any one of these N lines gives a particular value of T for that value of N and K. (These N lines should not be confused with the Q lines, which also curve down to the right. The Q lines give fabric weight in ounces per square yard; these are discussed later.)

K is called the reduced cover factor because N in equations (9) and (10) is the reduced yarn number of the fabric—i.e., the cotton yarn number of the hypothetical yarn whose diameter is the average of the warp and filling yarn diameters. It is obvious that this average diameter is D/2 since D is the sum of warp and filling yarn diameters:

$$D = d_1 + d_2. \tag{11}$$

If D is known from the main graph (obtained as described in the preceding section), the proper N line which relates K_w and T_w may be found quite simply by using the base scale in Figure 2 to locate the proper value of N opposite D/2.

The examples worked out previously in Table I are carried out in Figure 6 on the auxiliary graph by lifting just those portions necessary for illustration. In order to do this it was necessary to assign a value to D; the value of 15 mils was selected arbitrarily for the loom state. Reading the base scale in Figure 2, it can be seen that $N = 23$ lies opposite $D/2 = 15/2 = 7.5$ mils. The line $N = 23$ thus depicts the relation between K's and T's for the loom state. The flattened-state N value lies opposite $D_f/2 = (0.75)(15)/2 = 5.63$ mils, or $N = 41$. These lines are shown in Figure 6. It should be pointed out that if only crimps and threads per inch are known for a fabric, it is possible to define the fabric structure on the main graph and to locate the N line on the auxiliary graph. This N value, however, does not necessarily represent any kind of average of the warp and filling yarn numbers; it is merely the yarn number of a yarn with diameter equal to the average of the vertical diameters of the warp and filling yarns at the crossovers, and these latter are usually flattened.

It is also important to note that the diameter of the warp yarns alone, or of the filling yarns alone, can assume any value desired (i.e., $\beta = \sqrt{N_2/N_1}$ is not fixed), and that only their sum, D, is necessary to determine the relation between T and K. It can be observed by a little study of the unit cell cross section shown in Figure 1 that one yarn—say, the warp—can be made smaller in diameter while the filling yarn is made larger (keeping D constant) without changing the essential geometry of the unit cell, the only change being the movement of the point of tangency of the

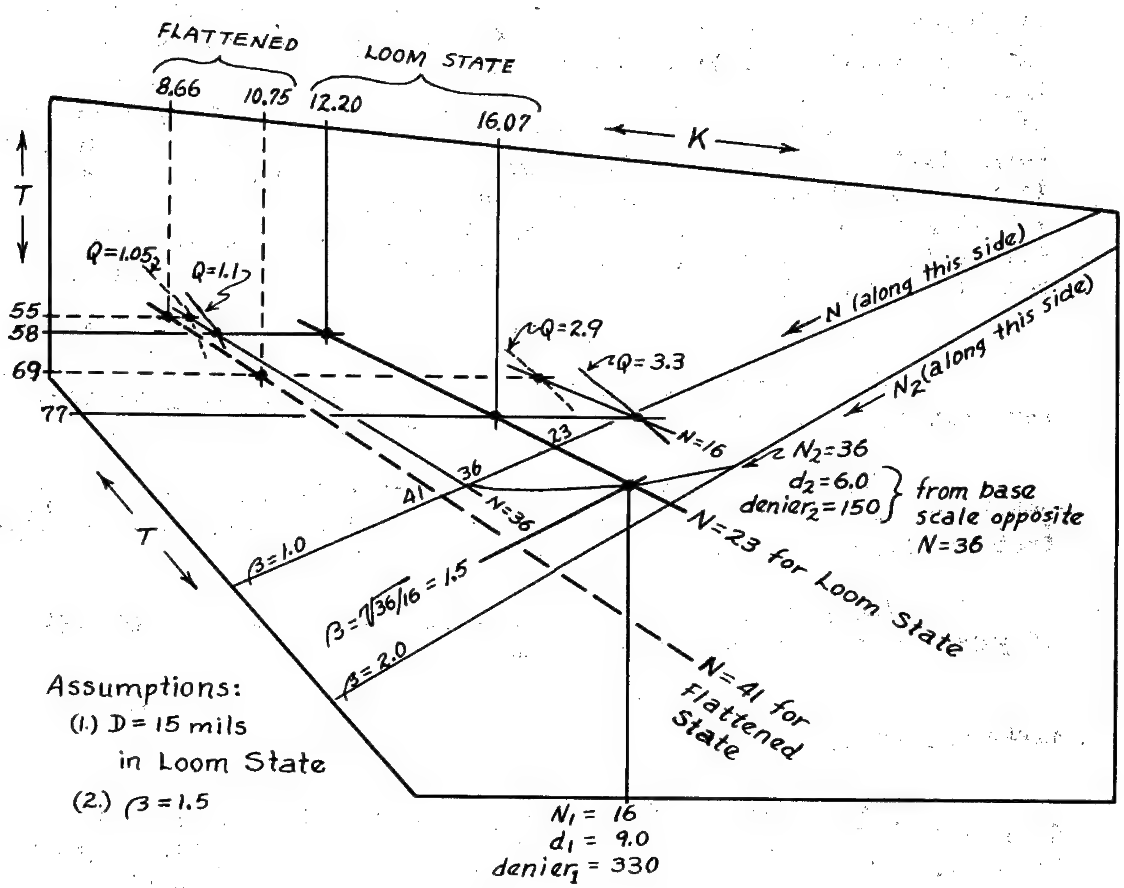


Fig. 6. Example of Use of Auxiliary Graph

NOTE: "N_1", "d_1", and "denier_1" could be expressed as "N_f", "d_f", and "denier_f"; and "N_2", "d_2", and "denier_2" as "N_w", "d_w", and "denier_w". The subscripts 1 and 2 are used merely to emphasize that the larger N value is always read along the $\beta = 2.0$ line and the smaller N value always along the base, regardless of which is warp and which is filling.

two yarns along the radius line defined by θ, which, along with all the other values. remains constant.

The reduced yarn number, N, is separated graphically into its two real components, N_w and N_f, by means of the set of divergent straight lines of various values of β running downward to the left from the upper right-hand corner of the graph (Figure 6), in conjunction with the base scale across the bottom of the graph giving yarn number, diameter, and denier. This operation is shown in Figure 6 for the loom-state example. The flattened state can also be separated into N_w and N_f, but the values for N_w and N_f thus obtained are not real because they represent the yarn numbers of round yarns having the diameter to which the real yarns have been flattened. Further, it is obvious that if the flattening is not proportional in each yarn, the β value will change. The diameter scale can be used for trial-and-error solutions for actual cases. The equation which relates the scales for diameter and yarn number is

$$d_w = 35.85/\sqrt{N_w}. \qquad (12)$$

The assumptions about yarn density which are necessary for this equation are discussed in the next section.

Various other dimensions obtainable from the auxiliary graph are shown in Table V (except for the h's, which were calculated from the h/D's given previously).

The fabric thickness is always given by whichever set of yarns gives the largest value of

$$G = h + d, \qquad (13)$$

where G = thickness in mils.

The fabric weight is obtained by the sum of the weight in warp and filling yarns. As indicated in Figure 6, the Q values should always be read at the intersection of the T lines and the sloping N lines which represent the actual yarn number regardless of diameter. The equations which are thus represented graphically are as follows:

$$\begin{aligned} W_w &= Q_w\ (1 + \%C_w/100) \qquad (14) \\ &= 0.686\ T_w\ (1 + \%C_w/100)/N_w \\ W_f &= Q_f\ (1 + \%C_f/100) \\ &= 0.686\ T_f\ (1 + \%C_f/100)/N_f \end{aligned}$$

$$\text{Total fabric weight} = W_w + W_f,$$

where W = weight in oz./yd.^{2} and Q = value read from graph.

Use of Graphs for Fibers Other Than Cotton

It should be emphasized that determination of flattened D_f from observed plain-weave thread spacings and crimps by the process described in a previous section (page 11) is valid for any type of flexible yarn, regardless of yarn density. The relations portrayed by the main graph are concerned with linear dimensions only. The yarn density is used to convert these linear dimensions to weights by relating yarn diameter and yarn number (or denier). This relation is as follows:

$$
\begin{aligned}
d_w &= 34.14/\sqrt{\phi_w \psi_w N_w} = 34.14/\sqrt{\rho_w N_w} \qquad (15)\\
&= 0.468\sqrt{\text{denier}_w/\phi_w \psi_w}\\
&= 0.468\sqrt{\text{denier}_w/\rho_w}\\
&= 35.85/\sqrt{N_w} \text{ for cotton,}
\end{aligned}
$$

where $\rho =$ yarn density $= \psi\phi$, $\psi =$ fiber density, and $\phi =$ packing coefficient.

Cotton yarns were assumed by Peirce to have $\rho = 0.9$ g./cc., which for $\psi = 1.54$ g./cc. gives $\phi = 0.589$, or 41.1% voids.

When equation (15) is substituted in the summation equation (11) for D:

$$
\begin{aligned}
D &= 34.14/\sqrt{\rho_w N_w} + 34.14/\sqrt{\rho_f N_f} \qquad (16)\\
&= (34.14)\ (1/\sqrt{N_w} + 1/\sqrt{N_f})\ /\sqrt{\rho}\\
&= (2)\ (34.14)/\sqrt{\rho N} = (2)\ (35.85)/\sqrt{N},
\end{aligned}
$$

where $2/\sqrt{N} = 1/\sqrt{N_w} + 1/\sqrt{N_f}$, $N =$ reduced yarn number as defined previously, and $\rho =$ average yarn density.

The reduced cover factor, K, is actually defined in terms of T and N (equation (10)), and, hence, in order to express it in terms of D (equation (8)), it is necessary to make certain assumptions about the yarn density. The value assumed by Peirce for cotton yarns $\rho = 0.9$, or, more precisely, $1/\sqrt{\rho} = \sqrt{v} = 1.0500$, where v is specific volume, cc./g.) was taken, which results in the constant 0.01395 used in equation (8). This assumption does not limit the use of the graph to cotton yarns and fabrics. The geometry of the fabric is the same, but the K values are in terms of cotton yarns which have the same diameter as whatever fabric or yarns of a given D are being considered. Thus, the proper name for the K used on the graph is "equivalent cotton plain-weave reduced cover factor." For example, a nylon fabric could have a

Table V

Fabric Dimensions Obtained From Auxiliary Graph

Fabric Structure	Yarn diameters (mils) W d_w	X F d_f	Ampli-tudes (mils) W h_w	X F h_f	Fabric thickness (mils) G	Reduced yarn No. N	Threads per inch W T_w	X F T_f	Weight ounces per square yard ($N_w = 36$; $N_f = 16$) W $Q_1(1 + C_1)$	X F $Q_2(1 + C_2)$	Total fabric
Loom state (minimum $\%C_f$)	6.00	9.00	7.50	7.50	16.50	23.0	58.3	76.8	1.34	3.65	4.99
Warp tension (minimum $\%C_w$)	6.00	9.00	4.50	10.50	19.50	23.0	70.0	66.9	1.40	3.79	5.19
"Normal crimps"	6.00	9.00	6.20	8.80	17.80	23.0	61.5	70.5	1.30	3.52	4.82
Swollen state (maximum)	6.60	10.00	6.90	9.70	19.70	18.6	66.0	74.0	1.46	3.97	5.43
Flattened state ($e = 0.75$)	4.52	6.73	5.63	5.63	12.36	41.0	55.1	68.7	1.13	3.08	4.21

reduced cover factor $K' = T/\sqrt{N}$, where N would be the cotton yarn number of the diameter-average yarn of warp and filling. This K' would not be usable in the main graph, since complete cover would not be given at $K' = 27.93$, but is given by $K = 27.93$. The proper K for the graph is obtained by inserting the actual D, in mils, in equation (8).

Fiber densities can be obtained from any textile handbook, but packing coefficients (or % voids in the yarn) must be determined by experiment for the particular yarns involved, or must be assumed on some rational basis. Peirce found that variations in packing density and fiber density tended to cancel out one another, and that therefore a standard yarn density of about 0.9 was a "reasonably good approximation for most textile materials." However, the entire shape of any woven structure is quite critically dependent upon the precise value of yarn density, and calculations of deviations from the simple ideal form, such as bowing, flattening, etc., require the use of the more precise value. It is unfortunate that handbooks do not list the % voids of different types of fibers in yarns of various twists.

Weave Density

In the concept of "weave density" outlined by Peirce, the swelling is allowed to continue until both sets of yarns jam each other and the structure is determined by λ, which is laid off along the jam line on the main graph. Although λ is not exactly linear along the jam line, the use of the interpolation scale gives good approximations for intermediate values.

The weave density of any fabric is readily obtained from the graph by use of the λ values. The weave density is obtained from the square of the value of the actual 1/D divided into the 1/D value which terminates on the jam line at the particular value of λ pertaining to the fabric. The relation is derived as follows:

$$D = 68.28/\sqrt{N}. \qquad (17)$$

Maximum D occurs when both threads jam. If N = constant, D_j at jam gives weave density, ρ_w:

$$(\rho_w = 68.28/ND_j^2) \div (\rho = 68.28/ND^2)$$
$$\therefore \rho_w = \rho D^2/D_j^2. \qquad (18)$$

But 1 is also constant; hence:

$$\rho_w = \rho\,(1/D_j)^2(D/1)^2 = 0.907(1/D_j)^2(D/1)^2, \qquad (19)$$

where ρ_w = weave density, g./cc., or density to which both yarns must swell to jam fabric; ρ = actual density, g./cc., of yarns = 0.907; $1/D$ is given; and $1/D_j$ = $1/D$ line which intercepts jam line at λ value of fabric.

<u>Example</u>: Assume original fabric $\lambda = 1.0$, $l_2/D = 1.45$

$$\therefore l_2/D_j = 1.047$$

$$\rho_w = (0.907)(1.047)^2/(1.45)^2 = 0.473 \text{ g./cc.}$$

<u>Example</u>: Assume original fabric $\lambda = 0.8$, $l_2/D = 1.00$

$$\therefore l_2/D_j = 0.93; \quad l_1/D = 1.25 \ (\therefore l_1/D_j = 1.16)$$

$$\rho_w = (0.907)(1.16)^2/(1.25)^2$$

$$= (0.907)(0.93)^2/(1.00)^2 = 0.782 \text{ g./cc.}$$

<u>Use of Graphs for Weaves Other Than Plain Weave</u>

As pointed out by Peirce, weaves other than plain weave contain the plain-weave unit cell at the yarn crossovers, and the graph for the plain weave can be used to depict these weaves by applying the proper correction factors to observed crimps and threads per inch. The correction factor is derived by adding the length of straight yarn in the float to the unit cell, and depends upon the type of weave and number of cross threads per float.

Basket (or Matt) Weaves.- The correction factor for basket weaves was given by Peirce, but is restated here in slightly more general terms to coincide with methods of calculation already presented herein. Primed quantities refer to the plain-weave equivalent:

$$T_w' = T_w/fR_w; \quad T_f' = T_f/fR_f \tag{20}$$

$$C_w' = C_w/R_f; \quad C_f' = C_f/R_w \tag{21}$$

where

$$R_w = \left[1 - \left(\frac{f-1}{f}\right)\left(\frac{X_w}{1000}\right)T_w\right],$$

$$R_f = \left[1 - \left(\frac{f-1}{f}\right)\left(\frac{X_f}{1000}\right)T_f\right],$$

f = number of cross threads per float, and X = center-to-center spacing, in mils, of threads not separated by cross yarns.

The quantity X may be measured with a magnifying micrometer in an actual fabric. It can often be assumed that $X \simeq d$, the yarn diameter, which, in turn, may be calculated for a circular yarn by using equation (15). However, the factor R is fairly sensitive to changes in X, and thus a value should be used which gives a reasonable degree of flattening as calculated from the equivalent plain-weave D.

Twill Weaves.- Only approximate analyses can be made for twill weaves, since an underlying assumption of the plain weave is that the projection of the threads into the plane of the fabric is a straight line. In the twill weaves the yarns tend to bow to one side at the float, which increases the crimp over what it would have been had the yarn been straight. Twill weaves can thus be studied by calculating this increase in crimp. Factors to convert to plain-weave equivalents are quite simple when it is assumed that the thread spacings are the same at crossovers and floats:

$$T_w' \simeq T_w; \quad T_f' \simeq T_f. \tag{22}$$

The amount of straight yarn in the float to be added to the crossover length depends upon the symmetry of the twill, and the correction factor for symmetrical twills (2/3, 3/3 types) is given:

$$C \simeq \frac{l - fp}{fp} \simeq \frac{l' + (f-1)p - fp}{fp}$$

$$\simeq \frac{l' - p}{fp} \simeq C'/f \tag{23}$$

$$\therefore C_w' \simeq fC_w; \quad C_f' \simeq fC_f.$$

For asymmetrical twills (2/1, 3/1 types) the correction factor is:

$$C \simeq \frac{l - \left(\frac{f+1}{2}\right)p}{\left(\frac{f+1}{2}\right)p} \simeq \frac{l' + \left(\frac{f-1}{2}\right)p - \left(\frac{f+1}{2}\right)p}{\left(\frac{f+1}{2}\right)p}$$

$$\simeq \frac{l' - p}{\left(\frac{f+1}{2}\right)p} \simeq \frac{C'}{\frac{f+1}{2}} \tag{24}$$

$$\therefore C_w' \simeq \left(\frac{f+1}{2}\right)C_w; \quad C_f' \simeq \left(\frac{f+1}{2}\right)C_f.$$

An equivalent plain-weave D can be calculated from these equations and used in equation (8) along with the equivalent plain-weave T's to obtain the two K values on the main graph which depict the plain-weave unit cell. If the observed twill crimps have been increased by bowing of the floats, this increase must be estimated and subtracted from the observed twill crimp before insertion in equations (23) or (24). The degree of flattening can then be obtained from the location of the plain-weave unit cell on the graph. Trial-and-error calculations will usually lead to reasonable values of bowing and flattening. This type of structure analysis must be accompanied by detailed microscope observations and a knowledge of the history of the fabric. For instance, the loom spacing of the warp in the reed has in some instances been used as a good estimate of the value of the plain-weave l for the filling yarns in a 3/1 twill.

APPENDIX

Basic Variables and Equations

Actually, the situation of eleven variables is not as bad as it sounds, since the group consists of five pairs in which one value is assigned to the warp yarns and the other to the filling yarns, plus the eleventh variable, D, the sum of diameters. The five variables are as follows: p, thread spacing, mils; l, thread length in unit cell, mils; h, thread amplitude, mils; C, fractional crimps ($100C = \%C$); θ, angle of inclination of thread, radians.

Peirce's analysis linked the eleven variables together with the following seven equations:

Projection parallel to cloth plane.-

$$h_1 = (l_1 - D\theta_1) \sin \theta_1 + D(1 - \cos \theta_1) \qquad (1)$$

$$h_2 = (l_2 - D\theta_2) \sin \theta_2 + D(1 - \cos \theta_2) \qquad (2)$$

Projection normal to cloth plane.-

$$p2 = (l_1 - D\theta_1) \cos \theta_1 + D \sin \theta_1 \qquad (3)$$

$$p1 = (l_2 - D\theta_2) \cos \theta_2 + D \sin \theta_2 \qquad (4)$$

Definition of crimp.-

$$C_1 = (l_1 - p2)/p_2 \qquad (5)$$

$$C2 = (l_2 - p_1)/p_1 \qquad (6)$$

Relation between paths of two sets of threads.-

$$D = h_1 + h_2 \qquad (7)$$

In his analysis, Peirce further simplified the situation of eleven variables by taking one of the variables, D, as a scale basis (or divisor) for three pairs of the other quantities: p, l, and h. However, no further simplification was possible, since no mathematical device could be found to eliminate and reduce the seven equations to one or two simple relations involving only thread spacings (p1 and p2), crimps (C_1 and C_2), and D. The best solution was the use of a table for p, h, and C for given intervals

of l and θ, but even with the table it is necessary to perform lengthy interpolations for any particular value. Peirce presented one graph and a graphlike device, but the former did not give precise intermediate values and neither one gave limit structures.

Construction of the Main Graph

A composite graph has been developed which depicts predise values for all of the variables as well as limit structures by using %C, crimp, versus T, threads per inch ($T = 1000/p$), as the basic coordinates, and superposing sets of lines for constant l/D, h/D, and θ. Actually, T is expressed in terms of K, the reduced cover factor, which will be explained later. The superposed lines are located as follows:

Constant l/D Lines

These lines are derived from the definition of crimp; the form of equation (5) may be rewritten in the following fashion:

$$C_1 = (l_1 - p_2)/p_2, \quad \text{or } 1/p2 = (1 + C_1)/l_1. \qquad (5)$$

Multiplying both sides by D:

$$(1 + C_1)D/l_1 = D/p_2 = T_2D/1000 = K_2/13.95, \qquad (8)$$

since, by definition, $1000/T = p$, K = the reduced cover factor.

If D/l_1 = constant (which is true along constant l/D lines), then the first and last terms in equation (8) reduce to:

$$K_2 = a + bC_1, \qquad (9)$$

in which a and b are constants. Thus, equation (9) shows that constant l_1/D lines are straight lines due to the linear relation between K_2, the reduced cover factor for one set of yarns, and C_1, the crimp in the other set of yarns. A similar situation exists between K_1 and C_2 for constant l_2/D lines. The subscripts are carried here to emphasize that the relation is between crimp in one set of yarns, "1," versus reduced cover factor (or threads per inch) in the other set of yarns, "2," or vice versa. Figure 1 shows these straight l/D lines on the $\%C_1$ versus K_2 graph.

Constant h/D Lines

The shape of constant h/D lines is that of approximate parabolas indicated by the approximate relation between $\%C_1$ and K_2 (or $\%C_2$ and K_1) derived by Peirce as follows:

Equations (1) and (3) were expanded to give:

$$h_1/D = 1.36 \sqrt{C_1}(p_2/D) \qquad (10)$$

$$= 136 \sqrt{\%C_1}/T_2D = 1.897 \sqrt{\%C_1}/K_2$$

(also a similar equation with reversed subscripts).

When h/D = constant:

$$K_2^{\,2} = a\%C_1, \text{ or } K_1^{\,2} = a\%C_2, \qquad (11)$$

in which a is an arbitrary constant, K is the reduced cover factor, and %C is the crimp.

Exact values of $\%C_1$ *versus* K_2 for plotting constant h/D lines were obtained by the use of Peirce's (1) table (Table II) relating values of h/D for various values of l/D and (l - p)/D. This was accomplished very easily by making an intermediate plot of h/D *versus* C for a family of constant l/D lines. It was then a simple matter to pick even values of h/D along these l/D lines and transfer them by means of the corresponding C value to the constant l/D lines on the $\%C_1$ *versus* K_2 graph. Smooth curves were then drawn through these points to give the constant h/D lines, as shown in Figure 2. The curves thus obtained give exact values of h/D.

Constant θ Lines

These are derived by substituting the definition of crimp, $l_1 = p_2(1 + C_1)$, and the definition of reduced cover factor, $13.95/K_2 = 1000/T_2D = p_2/D$, into equation (3) to give:

$$\%C_1 = (1 - \cos\theta_1)/\cos\theta_1 - (\sin\theta_1 - \theta_1\cos\theta_1)K_2/13.95\cos\theta_1 \qquad (12)$$

When θ = constant, this reduces to

$$\%C_1 = a - bK_2, \qquad (12a)$$

in which a and b are constants. These lines are shown in Figure 3.

Limit Structures

These are located as follows:

Jam Line: A thread is "jammed" when the straight portion $(l - D\theta) = 0$, as illustrated in Figure 4; it follows that:

Appendix Fig. 1

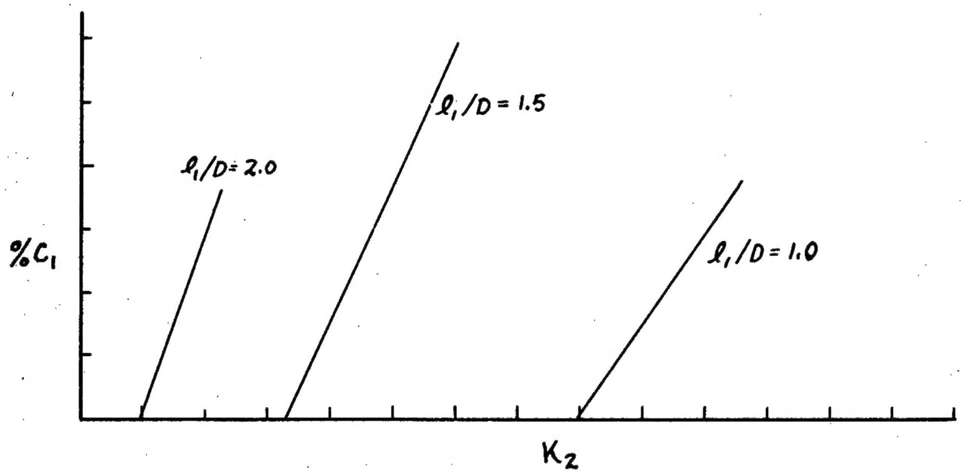
Constant ℓ/D Lines

$$(5.)\quad c_1 = (\ell_1 - p_2)/p_2 \;; \qquad \ell_1 = p_2(1+c_1)$$

$$(8.)\quad D/p_2 = T_2 D/1000 = \alpha K_2 = (1+c_1)\,D/\ell_1$$

If D/ℓ_1 = constant, then:

$$(9.)\quad K_2 = a + b\,c_1$$



Appendix Fig. 2

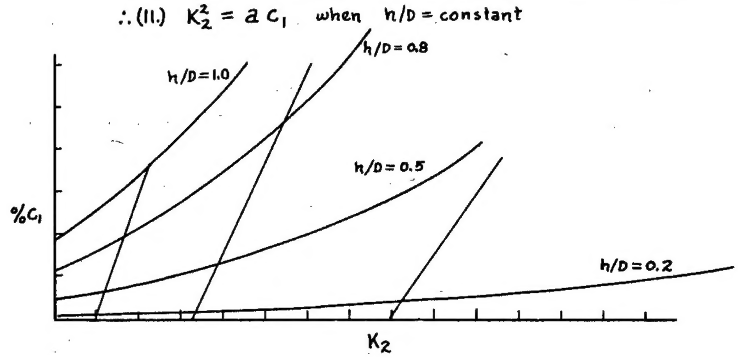
Constant h/D Lines

A. Peirce's table gives rigorous values of h/D in terms of ℓ/D and $(\ell-p)/D$. Since $c_1 = (\ell_1 - p_2)/D \div [\ell_1/D - (\ell_1 - p_2)/D]$, plot h_1/D vs. c_1 for constant ℓ_1/D lines and pick even values of h_1/D along ℓ_1/D lines to transfer to c_1 vs. K_2 graph.

B. Approximate parabolic form of constant h/D lines indicated by Peirce's expansion of equations (1.) and (3.) to give:

$$(10.)\quad h_1/D = 1.36\sqrt{c_1}\,(p_2/D) = 1360\sqrt{c_1}/T_2 D = 1360\sqrt{c_1}/\alpha K_2$$

$$\therefore (11.)\quad K_2^2 = a\,c_1 \quad \text{when } h/D = \text{constant}$$



Appendix Fig. 3

Constant θ Lines

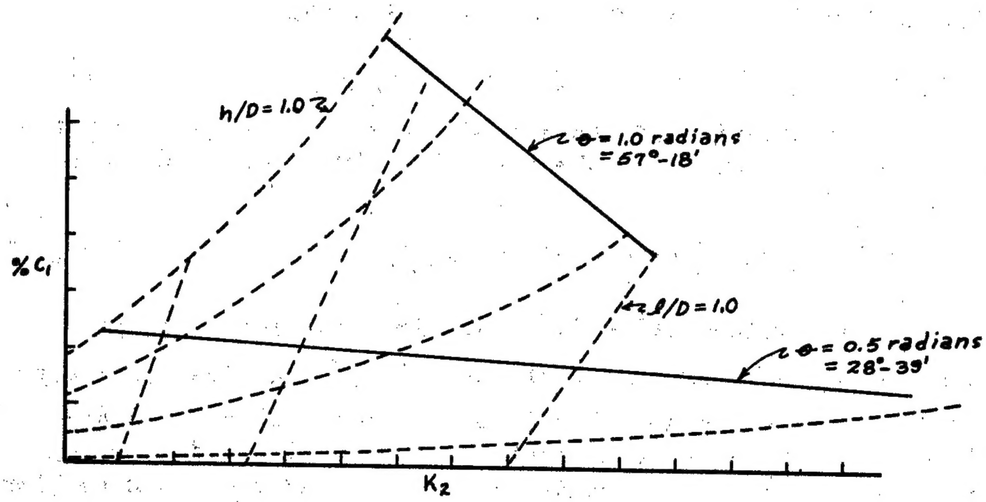
$$(3.)\quad p_2 = (\ell_1 - D\theta_1)\cos\theta_1 + D\sin\theta_1$$

Substituting: $p_2/D = 1000/T_2D = 1/\alpha K_2$

$$\ell_1 = p_2(1 + c_1)$$

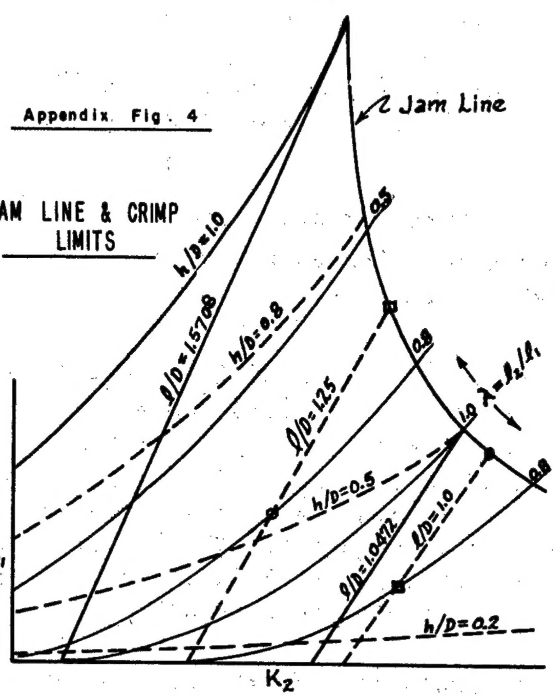
$$(12.)\quad c_1 = (1-\cos\theta_1)/\cos\theta_1 - \alpha K_2(\sin\theta_1 - \theta_1\cos\theta_1)$$

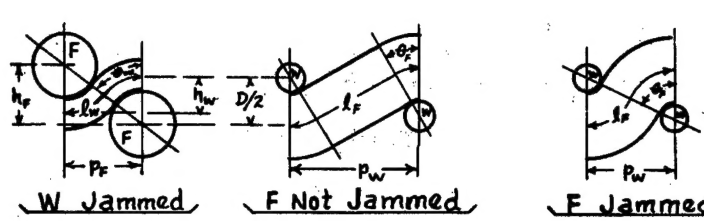
or, $c_1 = a - bK_2$ when θ = constant



Appendix Fig. 4

JAM LINE & CRIMP LIMITS





W Jammed — F Not Jammed — F Jammed

$$\theta_1 = l_1/D. \tag{13}$$

Also, equation (3) reduces to

$$p_2/D = \sin (l_1/D) = 1000/T_2D = 13.95/K_2, \tag{14}$$

in which subscript "1" refers to warp, W, and subscript "2" refers to filling, F.

Using the first two parts of equation (14) and substituting in the definition of crimp (equation (5)):

$$C_1 = [(l_1/D) - \sin (l_1/D)]/\sin (l_1/D). \tag{15}$$

Substitution of the second and fourth parts of equation (14) gives the equation of the "jam line" in terms of $\%C_1$ and K_2 only:

$$13.95/K_2 = \sin [(1 + C_1)13.95/K_2]. \tag{16}$$

Even values of $\lambda = l_2/l_1$ are located along the jam line by selecting even values of λ and of l_2/D to give particular values of l_1/D, which are then inserted in equations (14) and (15) to give points for the graph in terms of $\%C_1$ and K_2 which define the jam line. The "unit cell structure" mentioned earlier is defined by λ and either one of the l/D's.

Minimum Crimps (in F When W Is Jammed): Since the straight portion $(l - D\theta)$ of the warp thread equals zero, equation (1) reduces to:

$$h_1/D = 1 - \cos \theta_1 = 1 - \cos (l_1/D). \tag{17}$$

Combining this equation with equation (7) gives

$$h_2/D = \cos (l_1/D). \tag{18}$$

The particular values of l_1/D used above, which go with the proper even values of λ and l_2/D, were then used in Peirce's table of h vs. l and l - p to get precise values of C_2 to plot on the even-value lines of l_2/D. These points were then joined at equal values of λ to give the λ lines shown as solid lines curving down to the left in Figure 4. The l/D lines have been changed in Figure 4 from Figure 3 in order to show the ones which are related to limit points, such as $l/D = 1.5708$ and $l/D = 1.0472$.

Swollen Structure: Equations (19) and (20) show special cases of the swollen structure.

$$\theta_2 = l_2/D \tag{19}$$

$$\cos (l_1/D) + \cos (\lambda l_1/D) = 1 \tag{20}$$

Tightest Weave Containing Zero Crimp: Equations (21) and (22) show cases of the limit fabric which has zero crimp one way and maximum crimp the other—i.e., the W-jammed, F-straight fabric whose points lie at the opposite ends of $1/D = 1.5708$. A fabric with both points on an $1/D$ line less than 1.5708 cannot have zero crimp.

$$\theta_2 = 0 \qquad (21)$$

$$\theta_1 = l_1/D = 90^\circ = 1.5708 \text{ radians} \qquad (22)$$

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